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**Math 218** Quiz 1 (Edited) (March 2010)

 **Name**  ................................... ……….. **I.D**  .......................

**1.** (10%) Given (A|b) = 

Find the values of a so that the linear system AX=b has

1. no solution.
2. infinitely many solutions with 2 parameters.
3. infinitely many solutions with 3 parameters.
4. a unique solution

2a. (8 %) Find the inverse of the matrix  if it exists.

2b. (4 %) Prove that if A3 + 7A2 + 8A – 9 I = 0, then A – 1  exists.

3a. (5 %) Find det(A) if AAt = 2A and A = [5x5] .

3b. (7 %) Use elementary column operations to get two more zeroes

in the **4th column** in order to find the determinant of 

4a. (4 %) Use determinants to check whether the vectors $\left(\begin{matrix}3\\1\\2\end{matrix}\right)$, $\left(\begin{matrix}-1\\0\\2\end{matrix}\right)$, $\left(\begin{matrix}2\\1\\0\end{matrix}\right)$ form a basis for R3 .

4b. (7 %) Let a = (7 5 6 3 1 1)

 b = (0 2 1 4 3 1)

 c = (0 2 1 5 6 1)

Extend the subbasis {a,b,c} to a basis of R6 by the row-method (explained in class).

5a. (7 %) Consider the matrix A = 

Show that the vector b =  is a linear combination of the columns of A.

(Yes, we need an explicit answer like b= …C1+ .. C2 +…..+ …..).

5b. (7 %) Prove **one** of the following problems. **Choose one! Choose**  **one!**

* If u, v, w are linearly independent in Rn, then u, v, u+v+w are also linearly independent.
* If u, v, w are linearly independent in Rn and A=[n ×n] is an invertible matrix,

then Au, Av, Aw are also linearly independent.

1. (7 %) Prove **one** of the following theorems. **Choose one!**
* More than n vectors in Rn are linearly dependent. (Take 5 vectors in R4) (for simplicity)
* Fewer than n vectors in Rn cannot generate Rn. (Take 4 vectors in R5) (for simplicity)
* If the linear system AX=b has a unique solution, then the columns of A

are linearly independent.

7. (28 %) **True- False** & **more** as explained in the following box.

If False: **Make a good** **correction** underneath.

If True: Write True underneath . Do not justify

Penality: -1 for each wrong answer

1. A is an invertible n×n matrix$ ⟹$ AX = 2X has exactly one solution.
2. AB = AC and A $\ne $ 0 $⟹ $B = C.
3. rank(A) = 3 $⟹$ rank(A|B) = 4 or 5 or 6.
4. A = [4×7] and A $\ne $ 0 $ ⟹$ AX = b has a solution for all b in R4.
5. A = [7×4] and A $\ne $ 0 $ ⟹$ AX = 0 implies X = 0 only.
6. Any set of three vectors in R5 can be extended to a basis in R5.
7. Any set of 7 vectors in R5 are linearly dependent.
8. Any set of 5 elements in R5 form a basis of R5.
9. For any matrix A = [n×n] , the set {I, A, A2, ……, A7, A10,A11} are linearly dependent .
10. If A = [m×n] then AAT and ATA are both symmetric square matrices.
11. Suppose A=[m×n]. If the rows of A are linearly independent,

then the columns of A are linearly independent.

l.

m.

n.

8. (8 %) Let A = 

Fill in the blanks without proof.

1. rank(A) = ……………………
2. rank(A|A|A) = …………………….
3. In general, the linear system AX = b has a solution iff rank(A|b) = ……………….
4. rank(5A) – 5 rank(A) = ………………

*Good Luck*